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Singlet Interacting Neutrinos in the Extended Zee Model and Solar Neutrino Transformation

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Abstract

We study the impact of Standard Model singlet neutrinos on neutrino flavor transformation. We focus on an extension of the Zee model which includes singlet neutrinos, and find that the best limits on the interactions of the singlet neutrinos come from astrophysical phenomena. Singlet neutrino-electron scattering will impact both the matter enhanced flavor transformation potential as well as detector cross sections. If electron neutrino - singlet neutrino oscillations are responsible for the solar neutrino anomaly, then the limit on the singlet neutrino interaction strength is of order of the weak interaction scale. Zee model modification of $\nu_\tau - e$ scattering also impacts solar neutrino transformation, although this interaction is more tightly constrained.

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Recent data on atmospheric, solar and accelerator neutrino experiments [1] have indicated that neutrinos have a finite mass and they exhibit flavor oscillation phenomena. Although the data has yet to be confirmed they point to the possibility that the three known active neutrinos are insufficient to account for all observations. The mass squared differences required to fit the data are: $10^{-11} \text{ eV}^2 \leq \delta m_s^2 \leq 10^{-5} \text{ eV}^2$, $\delta m_a^2 \simeq 10^{-3} \text{ eV}^2$ and $0.2 \text{ eV}^2 \leq \delta m_{\text{LSND}}^2 \leq 2 \text{ eV}^2$ where the subscripts s , and a refer to solar and atmospheric oscillations respectively. At least one light standard model singlet fermion is required to reconcile the data with the standard two by two neutrino mixing explanations.

In most studies of a SM singlet neutrino one assumes it has no interactions other than gravitation at low energies and its role is to supply a mass for the active neutrinos. An example of a singlet neutrino is a right-handed neutrino of the Dirac type. Another example comes from SO(10) GUT model where the heavy SM singlet is a Majorana neutrino. In this model, the heavy neutrino has a mass of 10^{12-14} GeV and gives the active neutrinos a small mass via the seesaw mechanism. Its large mass prevents it from having direct experimental consequences at current available accelerator energies.

Recently there has been renewed interest in models of radiatively generated neutrino masses with new physics occurring at or not much above the weak scale. An example is the Zee model which generates neutrino mass with the same mechanism that produces lepton number violation [2]. The crucial ingredient is a lepton flavor changing SU(2) singlet scalar with U(1) hypercharge $Y=2$. It is straight forward to include a light SM neutrino in the model [3, 4], so as to accommodate all of the anomalous neutrino data. We observe that now this light singlet neutrino now has an interaction with a strength determined by the mass of the Zee scalar and its Yukawa couplings to the SM leptons. A detailed phenomenological study of the current limits on these parameters are given in Ref [4].

This paper is concerned with the effects of SM singlet neutrinos that have interactions as weak as or weaker than the normal weak interactions. These interactions can arise from a spin 1 particle exchange as in extended gauge models or a spin 0 boson exchange such as the charged scalar in the Zee model. The interactions may be weak due to small couplings and/ or a large mass of the mediating particle. One advantage of the Zee model is that it has relatively few parameters. Most are constrained by terrestrial experiments. This makes it a economical model for the study of singlet interacting neutrinos (SINs) on astrophysical phenomenon. Since there is now a large body of solar neutrino data with more to come, we look into the effects of SINs on some of the proposed solutions to the solar neutrino problem. For the purpose of this paper we can ignore the mixing of the Zee scalar with the requisite two Higgs doublets without loss of generality.

SINs can affect the study of the neutrino fluxes from the sun in two ways. They can impact on matter enhanced flavor transformation and secondly they alter the neutrino detector cross sections since they now interact with electrons albeit very weakly. The

interactions of the singlet will come into play in any scenario which involves the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [6] of neutrino transformation between an active neutrino and a singlet. We illustrate both effects with the small angle solution to the solar neutrino problem. If electron neutrinos transform to singlet neutrinos in the sun, the singlet neutrino coupling in the Zee model is most tightly constrained by the neutrino-electron scattering data at SuperKamiokande [5]. We also show that a model which produces neutrino mass, such as the Zee model, may provide additional interactions for the active neutrinos which will in principle occur in active-active transformation scenarios and detector cross sections. As we will demonstrate however, these parameters are constrained by terrestrial experiments to have a much smaller effect on neutrino flavor transformation.

We begin by looking at the general equation governing the transformation of electron neutrinos into another type of neutrino, where $x = \mu, \tau$, or s , and s is the singlet neutrino, in a matter environment. This is given by

$$i\hbar \frac{\partial}{\partial r} \begin{bmatrix} \Psi_e(r) \\ \Psi_x(r) \end{bmatrix} = \begin{bmatrix} \varphi(r) & \sqrt{\Lambda} + V_{ex}(r) \\ \sqrt{\Lambda} + V_{xe}(r) & -\varphi(r) \end{bmatrix} \begin{bmatrix} \Psi_e(r) \\ \Psi_x(r) \end{bmatrix}, \quad (1)$$

where

$$\varphi(r) = \frac{1}{4E} \left((V_e(r) - V_x(r))E - \delta m^2 \cos 2\theta_v \right) \quad (2)$$

$$V_e(r) \equiv \pm 2\sqrt{2} G_F \left[N_e^-(r) - N_e^+(r) - \frac{N_n(r)}{2} \right] \quad (3)$$

In these equations

$$\sqrt{\Lambda} = \frac{\delta m^2}{4E} \sin 2\theta_v, \quad (4)$$

$\delta m^2 \equiv m_2^2 - m_1^2$ is the vacuum mass-squared splitting, θ_v is the vacuum mixing angle, G_F is the Fermi constant, and $N_e^-(r)$, $N_e^+(r)$, and $N_n(r)$ are the number density of electrons, positrons, and neutrons respectively in the medium. In the formulas given here, upper signs (in this case plus) correspond to the mixing of neutrinos while the lower signs (in this case minus), correspond to the mixing of antineutrinos. The potential $\varphi_x(r)$ will have a standard model value and an additional term due to the extra interactions introduced by the Zee model. Other models which produce neutrino masses such as R-parity violating supersymmetric models may have similar interactions. Focusing on the Zee model, the charged scalar, h^- , is constrained by experiments at LEP to have a mass ($M_h > 100$ GeV). This in turn induces the following four fermion effective Lagrangian:

$$\mathcal{L} = \frac{|f_{12}|^2}{2M_h^2} \bar{\nu}_\mu \gamma_\mu \nu_{\mu L} \bar{e} \gamma^\mu e_L. \quad (5)$$

which governs low energy $\nu_\mu e$ scattering. Note that this term has the opposite sign as the SM charged current interaction and is a prediction of the model. This term has to be added to the standard model muon neutrino MSW potential as follows:

$$V_\mu(r) \equiv \mp 2\sqrt{2} G_F \left[\delta(N_{e^-}(r) - N_{e^+}(r)) + \frac{N_n(r)}{2} \right] \quad (6)$$

where $\delta = \sqrt{2}|f_{12}|^2/(8M_h^2 G_F)$. The coupling constant $|f_{12}|^2$ is constrained by the measurements of the lifetime of the muon to be $|f_{12}|^2/(\frac{M_h}{100\text{GeV}})^2 < .0015$ (see [4]). Therefore, $\delta < .002$. Similarly, $\nu_e \leftrightarrow \nu_\tau$ oscillations will be influenced by the Zee model via additional $\nu_\tau - e$ scattering contribution to the matter potential. The potential takes the form of Eq. 6 with the replacement of V_μ by V_τ and $\delta = \sqrt{2}|f_{13}|^2/(8M_h^2 G_F)$. The limit on $|f_{13}|^2$ is derived from LEP and SLC [7] measurements of the leptonic vector and axial vector couplings in Z decay. For a scale mass of $M_h = 800\text{ GeV}$; the limit is $\delta < 0.1$. Clearly the effect can be much larger for $\nu_e - \nu_\tau$ oscillation. This upper limit gives rise to a maximum change in the MSW potential of about 10%.

The extension of the Zee model which includes a singlet neutrino produces neutrino-electron scattering terms which, after Fierz transformation, have the form

$$\mathcal{L} = \frac{|g_1|^2}{2M_h^2} \bar{\nu}_R \gamma^\mu \nu_R \bar{e}_R \gamma_\mu e_R. \quad (7)$$

Here, g_1 is the Zee model coupling between the singlet neutrino, the right handed electron and the scalar. This produces a singlet neutrino MSW potential of

$$V_s(r) = \mp 2\sqrt{2} G_F \beta (N_{e^-}(r) - N_{e^+}(r)) \quad (8)$$

where

$$\beta = \left(\frac{\sqrt{2}|g_1|^2}{8M_h^2 G_F} \right). \quad (9)$$

The best terrestrial limit on β comes from the leptonic right handed coupling to the Z [4]. The singlet neutrino does not couple directly to the Z boson, it only makes a contribution to the decay through a correction at one loop order. The limit on $|g_1|^2/2M_h^2$ is about $2 \times 10^{-4} \text{ GeV}^{-2}$, therefore $\beta < 6$. Other than this one loop effect we found no direct experimental bound since this singlet neutrino can only enter in weak interaction processes via leptonic mixing and no usable constraint is available. This in principle allows the singlet-electron interaction to be larger than the weak interaction. However, as we shall see below astrophysical considerations can provide tighter constraints.

It is a characteristic of the Zee model that there are no interactions which convert electron neutrinos directly to muon, tau or singlet neutrinos by way of electron scattering mediated by the charged scalar. Therefore, in this model $V_{ex} = V_{xe} \approx 0$.

However, there are neutrino electron scattering terms in the Zee model which convert muon (tau) neutrinos to singlet neutrinos and vice versa. These are governed by the Lagrangian,

$$\mathcal{L} = -\frac{f_{12(13)}g_1^*}{2M_h^2} \left(\bar{\nu}_R \nu_{\mu(\tau)} \bar{e}_R e_L - \frac{1}{4} \bar{\nu}_R \sigma^{\mu\nu} \nu_{\mu(\tau)} \bar{e}_R \sigma_{\mu\nu} e_L \right) + h.c. \quad (10)$$

In the forward scattering direction, the scalar term is proportional to neutrino mass and can therefore be neglected. The tensor term is proportional to electron spin and integrates to zero for unpolarized electrons. Therefore, for most situations, $V_{\mu s(\tau s)} = V_{s\mu(s\tau)} = 0$.

Returning to the case of $\nu_e - \nu_s$ transition, the combined active-singlet neutrino transformation potentials can be cast in the form:

$$V_e - V_s = \pm \frac{3G_F N_N(r)}{\sqrt{2}} \left[\left(1 + \frac{2}{3}\beta \right) Y_e - \frac{1}{3} \right] \quad (11)$$

where N_N is the total number density of nucleons. The electron fraction is defined as

$$Y_e \equiv \frac{N_{e^-}(r) - N_{e^+}(r)}{N_N} \quad (12)$$

A non-zero β will have the effect of increasing the potential if the electron fraction is greater than $1/3$. In the sun for example, the electron fraction ranges from a value of about two thirds at the center to more than 0.85 in the outer layers. The main effect of the new potential is to change the position at which a given neutrino undergoes the MSW resonance. The position of the resonance is determined by the condition:

$$V_e - V_s = \delta m^2 \cos 2\theta_v. \quad (13)$$

Therefore, for given mixing parameters and a constant electron fraction, increasing β causes the resonance position for a neutrino of energy E to shift to lower density.

This scenario may be applied to several phenomenon, such as solar neutrinos and supernova neutrinos for the $\nu_e \leftrightarrow \nu_s$ or $\nu_e \leftrightarrow \nu_{\mu,\tau}$ situation. For the up-down asymmetry in the atmospheric neutrino problem, the oscillations between muon neutrinos and either tau neutrinos or singlet may be analyzed in a similar manner, although we note that in general there will be off-diagonal terms for $\nu_\mu \leftrightarrow \nu_\tau$ mixing.

Taking again the example of the sun, we see from Eq. 13 that all neutrinos will pass through the resonance condition at a position that is further from the center of the sun as β increases. For fixed δm^2 and $\sin^2 2\theta_v$, low energy neutrinos that in the case of $\beta = 0$ did not encounter resonances in the sun, will do so now if $\beta > 0$. We illustrate this point in Fig. 1 where the survival probability for solar neutrinos is plotted for two values of the parameter β . This figure was produced by numerically integrating Eq (1) for the singlet neutrino. For $\beta = 0$ the solution reduces to the small angle sterile neutrino oscillation

solution to the solar neutrino problem, as in given in, for example [8]. It can be seen that increasing β will cause a decrease in the number of low energy electron neutrinos coming from the sun.

In contrast, the nonzero β has little effect on the high energy neutrinos. The effective weak potential scale height

$$L_V = \left| \frac{d \ln(V_e - V_s)}{dr} \right|^{-1} \quad (14)$$

which determines the survival probability at the resonance position remains fairly constant with small changes in β . In fact for a fixed Y_e and an exponential density profile and a given neutrino energy, it can be shown that the weak potential scale height remains constant regardless of the value of β . Plots similar to Figure 1 can be drawn for $\nu_e \leftrightarrow \nu_{\mu,\tau}$ oscillations, although nonzero δ will have a smaller impact on the survival probability as the constraints on δ are tighter.

It is also important to take into account the effect of a non-zero β in neutrino detectors. In the Zee model, the neutrinos have only additional interactions with other leptons and not with the quarks due to the weak charge of the scalar. Hence, the radiochemical solar neutrino experiments such as SAGE and GALLAX [9, 10] *will not* be impacted by the additional interactions the singlet neutrino has. However, they will register the low energy neutrino flux which depends on β as explained before. On the other hand, an experiment which detects neutrino-electron scattering, such as SuperKamiokande however, will have some portion of its signal coming from singlet neutrino-electron scattering if singlet neutrinos are present. The cross section for singlet neutrino electron scattering at first order is given by:

$$\frac{d\sigma}{dT} = \frac{G_F^2 m_e}{2\pi} 4\beta^2 \quad (15)$$

where T is the electron recoil energy. In comparison, the largest contribution to the standard model $\nu_e - e$ scattering cross section approximately given by:

$$\frac{d\sigma}{dT} = \frac{G_F^2 m_e}{2\pi} \left[\left(1 + 2 \sin^2 \theta_W\right)^2 + (2 \sin^2 \theta_W)^2 \left(1 - \frac{T}{E}\right)^2 + \mathcal{O}(m_e/E) \right]. \quad (16)$$

The theoretical rate per electron recoil energy can be calculated by folding the survival probability for the electron neutrinos and the oscillation probability for sterile neutrinos with the cross sections and the fluxes of neutrinos. The detector rate may be estimated by folding the theoretical rate with an energy resolution function as in Eq 4 of Ref [11].

If $\nu_e \leftrightarrow \nu_s$ transformation is the solution to the solar neutrino problem, then it can be readily seen that a constraint on β comes from the overall number of events in Superkamiokande. Various combinations of mixing parameters and values of β will produce different total count rate. If any significant mixing of electron neutrinos and singlet neutrinos takes place in sun, and the flux predicted by the standard solar model [12] is correct, then β must be smaller than $\sim 1/2$. This limit is much stronger than the best limit from

accelerator experiments of $\beta < 6$ and is derived from the extreme situation where all electron neutrinos above 5 MeV are converted to singlet neutrinos.

Figure 2 plots the ratio of rates with to without matter enhanced flavor transformation, using the shape of the ^8B neutrino spectrum from [13]. Several curves are plotted representing several mixing parameters and values of β . It is seen that increasing the value of β causes a flattening of the recoil spectrum curve. It does not account for the upturn in event rate at high energy observed in the SuperKamiokande data [5].

Correctly reproducing the overall rates in all of the solar neutrino experiments with a singlet neutrino - electron scattering interaction, requires an adjustment to the usual sterile neutrino MSW mixing parameters. For example if $\beta = 0.3$, then δm^2 must be increased by $\sim 30\%$ (see Eq. 13) in order to avoid reducing the fluxes of pp neutrinos. The mixing angle must be adjusted to take into account both the effect of the change in δm^2 on the survival probabilities of the ^8B neutrinos and the effect of the nonzero singlet neutrino- electron scattering cross section in Kamiokande and Superkamiokande. For $\beta = 0.3$, $\sin^2 2\theta_v$ must be increased by $\sim 7\%$. The survival probability and expected electron recoil spectrum for these parameters is shown by the dot-dashed lines in Figures 1 and 2.

We turn to the case of $\nu_\tau - \nu_e$ oscillations. The $\nu_\tau - e$ scattering cross section can increase by a maximum of a factor of 2, depending on the neutrino and electron energy, since the Zee model amplitudes and the standard model amplitudes add coherently. The scattering cross section in this case looks like

$$\frac{d\sigma}{dT} = \frac{G_F^2 m_e}{2\pi} \left[\left(1 - 2\sin^2 \theta_W + 2\delta\right)^2 + (2\sin^2 \theta_W)^2 \left(1 - \frac{T}{E}\right)^2 + \mathcal{O}(m_e/E) \right]. \quad (17)$$

For $\delta = 0$, this reduces to the standard model neutral current cross section. We illustrate the effect for matter enhanced flavor transformation with parameters $\delta m^2 = 5.4 \times 10^{-6} \text{ eV}^2$ and $\sin^2 2\theta_v = 6.3 \times 10^{-2}$. In order to reproduce the observations in solar neutrino experiments with nonzero δ , the change in the MSW potential will force a maximum increase in δm^2 of 10% above the $\delta = 0$ solution. For this change in δm^2 the mixing parameter, $\sin^2 2\theta_v$ retains approximately its original value in order to take into account the both change in δm^2 and the increase in scattering at the detector.

For comparison we consider the case of vacuum oscillations. There is no change in the survival probability of electron neutrinos due to the singlet interactions. However, the electron recoil spectrum will be effected. The singlet neutrino - electron scattering gives a signal similar to the neutral current scattering, although the size of the effect depends on the unknown magnitude of β . Figure 3 shows the electron recoil spectrum for a vacuum solution of $\delta m^2 = 6.6 \times 10^{-11}$ and $\sin^2 2\theta_v = 0.9$. The active neutrino solution is very similar to the singlet neutrino solution with $\beta = 0.3$. Therefore in the vacuum case, as in the small angle case, it may be difficult to differentiate between singlet neutrino oscillations and active neutrino oscillations just by detecting neutrino-electron scattering.

On the other hand, SNO [15] may be able distinguish the active-singlet solution from both the active-active oscillation and the active-sterile oscillation through the combination of the three reactions: (1) $\nu_e + d \rightarrow p + p + e^-$, (2) $\nu_x + d \rightarrow p + n + \nu_x$ and neutrino-electron scattering (3) $\nu_x + e^- \rightarrow \nu_x + e^-$. In reaction 2, ν_x can be ν_e , ν_μ , and ν_τ . In reaction 3, ν_x includes the three active neutrinos and also the singlet neutrino. The expected event rate per kilotonne year from the three reactions may be found in [15]. The standard solar model predicts 6500 events from reaction 1 above a 5 MeV threshold, and about 2200 will be seen for vacuum $\nu_e \leftrightarrow \nu_\tau$ oscillations. These numbers remain roughly unchanged in the presence of a singlet or sterile neutrino, although there will be some variation depending on the choice of mixing parameters. The number of events for reaction two is 710 from the standard solar model and remains unchanged in the case of active-active oscillations, although it should be reduced by about a factor of two in the case of active-sterile or active-singlet oscillations, depending on the mixing parameters. From reaction 3, about 320 events are expected for active-active oscillations with about 78 coming from $\nu_{\mu(\tau)}$. The number of neutrino-singlet interaction events depends on the strength of the interaction β . Therefore, if reaction 2 indicates sterile or singlet neutrinos, for large β reaction 3 in combination with 1 must be used to distinguish the two cases.

In conclusion, we find that the extended Zee model with its new lepton number violating interactions gives rise to new neutrino-electron scattering mechanisms. The additional scattering occurs for muon, tau and singlet type neutrinos. It alters the small angle MSW $\nu_e - \nu_s$ and $\nu_e - \nu_\tau$ solar solution by shifting the resonance position for neutrinos of a given energy. Furthermore for it increases the number of counts in water detectors and modifies the shape of the electron recoil spectrum produced from neutrino electron scattering. Taking these into account we have obtained a limit on singlet neutrino - electron interaction β which is an order of magnitude better than derived from current accelerator experiments. With non-zero Zee model interactions for the singlet and tau neutrinos, the apparent δm^2 as measured by solar neutrino experiments can differ by as much as $\sim 50\%$ and $\sim 10\%$ respectively, from that which would be measured by reactor experiments.

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Figure 1: Survival probability for electron neutrinos produced near the center of the sun is plotted against neutrino energy. The solid line is for electron neutrino-singlet neutrino mixing parameters of $\delta m^2 = 4 \times 10^{-6} \text{eV}^2$, $\sin^2 2\theta_V = 0.01$ and a singlet neutrino interaction parameter, $\beta = 0$. This corresponds to the case of sterile neutrinos. The dashed line is for $\delta m^2 = 4 \times 10^{-6} \text{eV}^2$, $\sin^2 2\theta_v = 0.01$, $\beta = 0.3$ and shows that increasing the value of β causes less electron neutrinos to survive. The dot-dashed lines represents $\delta m^2 = 5.2 \times 10^{-6} \text{eV}^2$, $\sin^2 2\theta_v = 0.011$, $\beta = 0.3$. The solid and dot-dashed lines will produce very similar signals in current solar neutrino detectors.

Figure 2: The ratio of the number of electrons produced by neutrino electron scattering to the predicted number of electrons using an undistorted ^8B spectrum in a detector such as SuperKamiokande is shown. The solid, dashed and dot-dashed lines correspond to the same parameters as in Figure 1. Increasing β causes a flattening of the curve and overall increase in the number of counts.

Figure 3: Shows the same ratio is as in Figure 2 for vacuum oscillations. The solid line corresponds to vacuum oscillations into active neutrinos, where the dashed line corresponds vacuum oscillations into singlet neutrinos with interaction parameter $\beta = 0.3$.





